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SOLUTION BY H. S. UHLER, Yale University.

Since the surfaces are figures of revolution we may confine our attention to a parabola with its axis vertical and a co-planar circle whose center lies in this axis. Let the coördinates of the center of the circle be $(0, h)$. The equation of the circle is

$$x^2 + (y - h)^2 = r^2 .$$

Elimination of x between this equation and $x^2 = 4ay$ gives

$$y^2 - 2(h - 2a)y - (r^2 - h^2) = 0.$$

In order that the sphere may touch the paraboloid the last equation must have equal roots, the necessary and sufficient condition being

$$4(h - 2a)^2 + 4(r^2 - h^2) = 0, \quad \text{or} \quad h = (r^2 + 4a^2)/4a.$$

The volume of the paraboloid up to the level $h + r$ is given by

$$\pi \int_0^{h+r} x^2dy = 4\pi a \int_0^{h+r} ydy = 2\pi a(h + r)^2 = \frac{\pi(r + 2a)^4}{8a}.$$

In general, therefore, the volume of the water is expressed by

$$v \equiv \frac{\pi(r + 2a)^4}{8a} - \frac{4\pi r^3}{3}.$$

Two cases now present themselves, namely, (i) a is kept constant while r varies, or (ii) r is fixed in value while a changes.

Case (i)

$$\frac{\partial v}{\partial r} = \frac{\pi[r(r - a)^2 + 11a^2r + 8a^3]}{2a}.$$

Since this expression cannot vanish for any positive value of r the necessary condition for maxima and minima is not fulfilled, as is also apparent from general considerations.

Case (ii)

$$\frac{\partial v}{\partial a} = \frac{\pi(r + 2a)^3(6a - r)}{8a^2},$$

and

$$\frac{\partial^2 v}{\partial a^2} = \frac{\pi}{4a^3}[3a(r + 2a)^3 + (6a - r)(a - r)(r + 2a)^2].$$

When $6a - r = 0$, the first and second derivatives become 0 and $+ 64\pi r$, respectively. Consequently, when $a = r/6$ a minimum of volume is attained. It should be remarked, however, that $h = 5r/3$ and $h - r = 2r/3$, so that the lowest point of the sphere is two-thirds of the radius above the vertex of the paraboloid; in other words, the sphere has not literally been "dropped to the bottom."

Also solved by A. W. SMITH, F. A. POTTLE, HORACE OLSON, J. W. CLAWSON, C. A. NICKLE, O. S. ADAMS, PAUL CAPRON, J. A. BULLARD, GEO. W. HARTWELL, J. A. CAPARO, A. G. RAU, H. C. FEEMSTER, and G. PAASWELL.

404. Proposed by B. J. BROWN, Victor, Colorado.

Solve the differential equation, $(x^2 - y^2)(1 + dy/dx) = 2xy(1 - dy/dx)$.

SOLUTION BY H. L. AGARD, Williams College.

The equation may be written,

$$(x^2 - 2xy)dx + (2xy - y^2)dy + x^2dy - y^2dx = 0,$$

which, after adding and subtracting x^2dx and y^2dy, becomes,

$$2x(x - y)dx + 2y(x - y)dy - (x^2 + y^2)(dx - dy) = 0.$$

Introducing the integrating factor $(x-y)^{-2}$, we have,

$$\frac{(x-y)(2xdx+2ydy)-(x^2+y^2)(dx-dy)}{(x-y)^2}=0,$$

which, upon integration, becomes

$$\frac{x^2+y^2}{x-y}=c, \qquad \text{or} \qquad x^2+y^2-c(x-y)=0.$$

Also solved by A. W. Smith, Norman Anning, J. W. Clawson, J. A. Bullard, G. Paaswell, O. S. Adams, Elijah Swift, Frederick Wood, Horace Olson, C. A. Barnhart, L. M. Coffin, G. W. Hartwell, J. D. Bond, A. G. Rau, C. A. Hutchinson, Claribel Kendall, C. S. Atchinson, J. W. Cromwell, C. P. Sousley, J. A. Caparo, and Paul Capron.

405. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

Determine the greatest quadrilateral which can be formed with the four given sides a, b, c, and d taken in order.

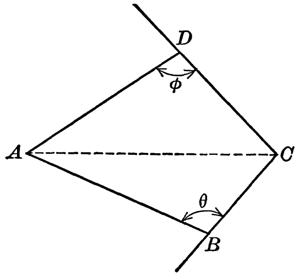
Solution by A. M. Harding, University of Arkansas.

In the quadrilateral $ABCD$, if $AB=a$, $BC=b$, $CD=c$, $AD=d$, we have

$$\overline{AC}^2=a^2+b^2-2ab\cos\theta=c^2+d^2-2cd\cos\phi.$$

Hence,

$$ab\cos\theta-cd\cos\phi=\frac{a^2+b^2-c^2-d^2}{2}. \tag{1}$$



Also, area $=\frac{1}{2}ab\sin\theta+\frac{1}{2}cd\sin\varphi$. For a maximum or minimum we must have

$$ab\cos\theta d\theta+cd\cos\varphi d\varphi=0. \tag{2}$$

From (1) we obtain, by differentiation,

$$-ab\sin\theta d\theta+cd\sin\varphi d\varphi=0. \tag{3}$$

It follows from (2) and (3) that

$$\tan\phi=-\tan\theta; \qquad i.e., \qquad \phi+\theta=180^\circ. \tag{4}$$

Therefore,

$$\cos\theta=\frac{a^2+b^2-c^2-d^2}{2(ab+cd)}.$$

Hence, the given quadrilateral will be a maximum when it can be inscribed in a circle.

Note.—It is evident from the nature of the problem that it is necessary to consider only *convex* quadrilaterals, and that (4) gives a maximum and not a minimum.

406. Proposed by C. N. SCHMALL, New York City.

Given $f(x+h)+f(x-h)=f(x)\cdot f(h)$, determine by Taylor's theorem or otherwise the nature of the function f.

Solution by W. M. Carruth, Hamilton College, New York.

Given

$$f(x+h)+f(x-h)=f(x)\cdot f(h). \tag{1}$$

Put $h=0$ in this equation. Then $2\cdot f(x)=f(0)\cdot f(x)$. Hence, either

$$f(x)=0, \tag{2}$$